Pricing Asian Options

The price of a stock changes from day to day. The price of a stock on a particular day is a function of the price of a stock on the previous day. The list of stock prices for a number of consecutive days is called its path. A stock price path can be simulated using the formula

$$S_{t+1} = S_t e^{\alpha+\sigma Z_t}$$

where Z_t is a Normal$(0, 1)$ random variable, or a standard normal. Given an initial stock price S_0 and values for α and σ, it is possible to simulate a stock price for a certain number of days.

(a) Write a function that takes S_0, α, σ, and T, the number of time points, and returns a stock price path of length T.

(b) Use $\alpha = 0.002$, $\sigma = 0.01$, and $S_0 = 100$ to simulate and plot a stock price path of 90 days. Save the figure as "onepath.pdf"

(c) Determine a Monte Carlo estimate of the stock price after 90 days and assess Monte Carlo error.

(d) Plot the simulated density function of the stock prices at 90 days. Save the figure as "stock-density.jpeg"

A call option is a contract that can be purchased. A call option gives the owner the right, but not the obligation, to buy a stock for a fixed price, K, at a date in the future, T. For example, a call option could be the right to buy a stock in 3 months for K =\$100, regardless of what the actual price of the stock is. If the price of a stock is \$120 at time T, then the option essentially makes the owner \$20 because they can buy the stock for \$100 and then sell it for \$120 and make \$20. If the stock costs \$80 then the option is worth nothing because the owner of the call option won't spend \$100 to buy something that only costs \$80. The option pay out is a function of the stock price at time T, which is S_T. It can be written as

$$max(0, S_T - K).$$

The price of a call option can be determined using simulation. The method is to simulate many stock price paths, find the option pay out at time T, and average all the option payouts.

(e) Use a Monte Carlo study to determine the price of a call option. Use the same situation from part b and use a strike price of $K = 110$. Asses the Monte Carlo error.

(f) Write a function that takes as inputs S_0, α, σ, T, and K that outputs the Monte Carlo estimate and the Monte Carlo error of the call option price.

It turns out that mathematical finance concepts can be used to determine exact values for stock prices and call option prices in the previous scenarios. An Asian call option, however, can only be priced using a simulation study. For an Asian call option, instead of using the price at time T to compare against the strike price, the average of the entire path is used. In this case, the payout of the option is

$$max(0, A_T - K),$$

where $A_T = \frac{1}{T}\sum_{i=1}^{T} S_i$

(g) Use a Monte Carlo study to determine the price of an Asian call option. Use the same situation from part b and use a strike price of $K = 110$. Asses the Monte Carlo error.

(h) Write a function that takes as inputs S_0, α, σ, T, and K that outputs the Monte Carlo estimate and the Monte Carlo error of the Asian call option price.

(i) Use the situation from part b and the functions you wrote in parts f and h. Determine the price of the call option and Asian call option for strike prices ranging from 80 to 140, incrementing by 5. Plot these prices against K on the same figure. Name the figure "bystrike.pdf".