

# Simulation Study of Polya Urn

## 1 Introduction

Deep within the annals of history, the civilization of Polya left behind an intriguing artifact: a mystical urn. According to Polya's legends, this wasn't just any urn - it had peculiar properties. When a ball was drawn from this urn, it demanded that two balls of the same color be returned. The urn was considered a key component in royal ceremonies, with the color of balls symbolizing the prosperity of the coming year. An even number of black and white ball in the urn was a sign of immense fortune, while a higher proportion of black balls portended challenging times.

Recent archaeological excavations unearthed one of these legendary urns. Initial observations showed a mix of black and white balls. To decode its mysteries, a preliminary experiment was conducted. After drawing 100 balls from the urn, each time returning two of the same color, it was observed that 75 of them were black.

At this point in the experiment, you wonder if there are an equal number of black and white balls. Your task is to use the simulation method to investigate this hypothesis further.

## 2 Assignment

You are required to set up a Monte Carlo simulation study to address the following:

- Given the peculiar property of Polya's urn, develop a simulation where you model the process of drawing a ball and then returning two balls of the same color.
- Using the simulation, test the null hypothesis that 50% of the balls are black against the one-sided alternative hypothesis that more than 50% of the balls are black. Under the null hypothesis, the urn begins with 10 balls, 5 of each color.
- Calculate the p-value for the observed test statistic of 0.75 (75 black balls out of 100).
- If the level of significance is 0.05, determine the rejection region for this problem.
- For an alternative proportion of black balls of 0.6, calculate the power of the test. Repeat using alternative proportion of black balls being 0.7.
- Be sure to assess the Monte Carlo error in your results.

Note: You should not use built-in statistical tests or libraries that directly calculate power or p-values. The aim is to rely on the simulation process for your conclusions.