$$X_i \overset{iid}{\sim} \text{Gamma}(\alpha, \beta)$$ (shape, rate)

$$p(\underline{x}) = p(x_1, \ldots, x_n) = \prod_{i=1}^{n} p(x_i)$$

$$= \prod_{i=1}^{n} \frac{\beta^{\alpha}}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-\beta x_i}$$

$$L(\alpha, \beta) = \beta^{n\alpha} \left[\Gamma(\alpha)\right]^{-n} \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} e^{-\beta \Sigma x_i}$$

$$\ell(\alpha, \beta) = \log L(\alpha, \beta) = n\alpha \log \beta - n \log \Gamma(\alpha) + (\alpha-1) \sum_{i=1}^{n} \log(x_i) - \beta \Sigma x_i$$

$$\nabla \ell(\alpha, \beta) = \left(n \log \beta - n\,\Psi(\alpha) + \sum_{i=1}^{n} \log(x_i), \; \frac{n\alpha}{\beta} - \Sigma x_i \right)$$

where $\Psi(z)$ is the digamma function

$\Psi_1(z)$ is the trigamma function

$$D^2 \ell(\alpha, \beta) = \begin{bmatrix} -n\,\Psi_1(\alpha) & \frac{n}{\beta} \\ \frac{n}{\beta} & \frac{-n\alpha}{\beta^2} \end{bmatrix}$$