Tied-Up-Normal Model : $X_1, \ldots, X_m$ are iid $N(\theta, \theta)$ ← variance

$$L(\theta \mid x_1, \ldots, x_m) = \prod_{i=1}^{m} (2\pi)^{-\frac{1}{2}} \theta^{-1/2} \exp\left(-\frac{1}{2\theta} (x_i - \theta)^2 \right)$$

$$\ell(\theta \mid x_1, \ldots, x_m) = \log L(\theta \mid x_1, \ldots, x_m)$$

$$= -\frac{m}{2} \log(2\pi) - \frac{m}{2} \log \theta - \frac{1}{2} \theta^{-1} \sum_{i=1}^{m} (x_i - \theta)^2$$

$$\frac{d\,\ell(\theta \mid x_1, \ldots, x_m)}{d\theta} = -\frac{m}{2}\theta^{-1} + \frac{1}{2}\theta^{-2} \sum_{i=1}^{m} (x_i - \theta)^2 + \theta^{-1} \sum_{i=1}^{m} (x_i - \theta)$$

$$\frac{d^2 \ell(\theta \mid x_1, \ldots, x_m)}{d\theta^2} = \frac{m}{2}\theta^{-2} - \theta^{-3} \sum_{i=1}^{m} (x_i - \theta)^2 - \theta^{-2} \sum_{i=1}^{m} (x_i - \theta) - m\theta^{-1}$$

$$- \theta^{-2} \sum_{i=1}^{m} (x_i - \theta)$$

$$= -m\theta^{-1} + \theta^{-2} \left[\frac{m}{2} - 2 \sum_{i=1}^{m} (x_i - \theta) \right] - \theta^{-3} \sum_{i=1}^{m} (x_i - \theta)^2$$