Stat 624: Midterm

Put all completed files inside your git repository in the folder titled midterm. Push all files to submit. To ensure you will get full credit, ensure the problems are all clearly labelled, even parts a, b, c, etc.

Problem 1

Put all solutions for this problem in a file labeled Problem1.R inside the midterm folder. The Frechet distribution is sometimes used for extreme value problems becasuse it has a thicker tail than other distributions. It is a three parameter distribution, but if we assume a couple parameters, the density function can be written as

$$f_X(x) = \alpha x^{(-1-\alpha)} e^{-x^{-\alpha}}$$

where $\alpha > 0$ is the only parameter and the domain is $x > 0$.

a Theoretically, the mode of a Frechet distribution is $\left(\frac{\alpha}{\alpha+1}\right)^{1/\alpha}$. Check this using a Newton-Raphson algorithm to calculate the mode of the density function. Check your solution for $\alpha = 3$.

b Use the bisection method to determine which value of α will return a mode of 0.8. You can use the theoretical mode formula instead of your Newton Raphson algorithm.

c Plot the mode as a function of α. Indicate in the plot the value from the last problem where the mode was equal to alpha. Provide the code used to create this plot and save it as "mode.pdf"

d The CDF is $F_X(x) = e^{-x^{-\alpha}}$. Use this to write a function that will simulate a given number of random draws from a Frechet distribution. Have the number of draws and α be inputs for the function and return an error if an invalid value of α is given.

e A machine tests many radioactive containers to detect how much leakage there is. Anything under 2 it does not detect but anything above 2 it flags. 140 containers were tested and 5 containers had leakage above 2. Suppose that leakage follows a Frechet distribution and it is believed that the parameter value for α is 5. Using the results of the experiment, test the hypothesis that $\alpha = 5$ using a simulation study where the alternative hypothesis is $\alpha \neq 5$. Report the p-value and assess Monte Carlo error.

f A better machine was brought in and all 140 containers were tested again. The results are in the file "leaky.csv". Find the maximum likelihood estimate of α. Built in optimization functions in R or Python are okay to use (I already made you use Newton-Raphson once!). Assess the uncertainty of your estimate with a bootstrap confidence interval.

Problem 2

Put all solutions for this problem in a file labeled Problem2.R inside the midterm folder. Consider the following integral in the problems below

$$\int_0^{2\pi} \sin(mx)\sin(nx)dx$$

a Write a function that inputs values for m and n and returns the integral using numerical integration using either simpson's rule integration or trapezoidal rule. Check your function for $m = n = 1$ and for $m = 2; n = 3$.

b Write a function that performs Monte Carlo integration for any values m and n. Check your function for $m = n = 1$ and for $m = 2; n = 3$. Choose B to be high. enough so that the answer to this part is similar to the previous one, especially for smaller values of m and n (1, 2, or 3).

c Test several other values for m and n including cases where $m = n$. What pattern do you see? (Hint: this is a pi day themed problem!!)